

## **Controlling Harmonics in the Polyphase Induction Motor**

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**Abstract: The basic theory and design of the polyphase induction motor is reviewed and the nature of the individual harmonic motor fields is derived from theory. Characteristics of the individual harmonic fields, and earlier methods of harmonic field control are discussed. New data is presented to show that phase balancing with the USES Unit, reduces the dominant 5th harmonic stator field by 50%. This significant improvement in control of retarding motor harmonics should lead to improved motor performance and circuit design.**

**Key Words: phase balancing, induction motor, harmonic fields, motor performance, circuit design**

At Northrop Grumman Corporation's (NGC) Sykesville, Maryland facility Cutler-Hammer Engineering Systems Services, Inc. (CHESS) conducted a series of tests to determine the effect of a Magnetic Phase Balancing Device, known as the USES Unit, on the performance of a 150 HP induction motor, [1] [2] [3]. This instant paper is a discussion of the sources and effects of the harmonics fields in the induction motor and the control of those harmonic fields.

The polyphase induction motor as designed by Tesla [4] and Steinmetz [5], as indicated in Fig. 1, consists of two concentric cylinders: the stator and the rotor, which contain embedded conductors. The rugged simplicity, flexibility of design and low maintenance has made this brushless motor the workhorse of modern industry and there are more than 100 million in use in the USA today. The development of mathematical models, improved materials and the intense pressure for lower costs has advanced the art of motor design. As a result, the horsepower rating at 1800 rpm of the induction motor supported by a NEMA 404 frame has increased from 7.5 to 70 HP over the period from 1898 to 1970. The design of the windings of these motors has not changed significantly in the past 50 years.

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## I. Review of Motor Theory.

Ampere's Law (1) states that if a current carrying conductor is placed in a magnetic field, a useful force may be obtained in a direction perpendicular to both,

$$F = ILB \quad (1)$$

where,  $F$  is force,  $I$  is current,  $L$  is length of the conductor and  $B$  represents the magnetic field strength. The magnetic field is produced by the stator coil windings.

For the stator, to produce the magnetic field utilizing 3-phase power, three individual windings, each excited by a separate voltage source and separated by 120 electrical degrees, produce a magnetic force wave that rotates around the air gap at a synchronous speed, defined as the cycles per second/# magnetic poles in the stator. These rotating magnetic fields induce voltages and therefore currents into the closed rotor bars according to Faraday's Law (2),

$$e = BLv \quad (2)$$

where:  $e$  = voltage,  $B$  = field strength,  $L$  = length of conductor, and  $v$  represents the velocity of movement.

When a rotating magnetic flux is generated by the stator windings, a voltage is induced into the rotor bars, which causes a current to flow with an associated magnetic field, according to (1). These balanced, opposing magnetic fields result in tangential forces, which cause useful rotation, or motor action.

To visualize the polyphase induction motor, a thorough understanding is required of the way currents flow in the distributed stator coil windings to produce the rotating magnetic fields. In the stator and the rotor, as shown in Fig. 1, wires are imbedded in slots to form the required very narrow air gap and to reduce forces acting on the conductors.

In Fig. 1 there are 9 stator slots per pole, and the electrical angle between slots is 20 degrees, which will be the phase displacement of the successive coil voltages. With uniform speed of rotation, therefore, the space-phase angles on the periphery and in the magnetic fields are interchangeable with the time-phase angles in the electric circuits, and the same phasor diagram may represent either space or time relations. In order to achieve the rotating magnetic field wave, the coils of the 3-phase stator winding are assembled as shown in Fig. 2, which shows 1/2 the winding or 2 poles and 16 slots.

This intricate pattern of stator windings, required to project the magnetic field across the narrow air gap, is the source of the dominant phase-belt 5<sup>th</sup>, and 7th harmonics. Here-to-fore these have been accepted as a characteristic of the machine, a cost of doing business. In the data section of this paper a new method of controlling these harmonics will be presented, which was not available during the intensive motor winding design phase.

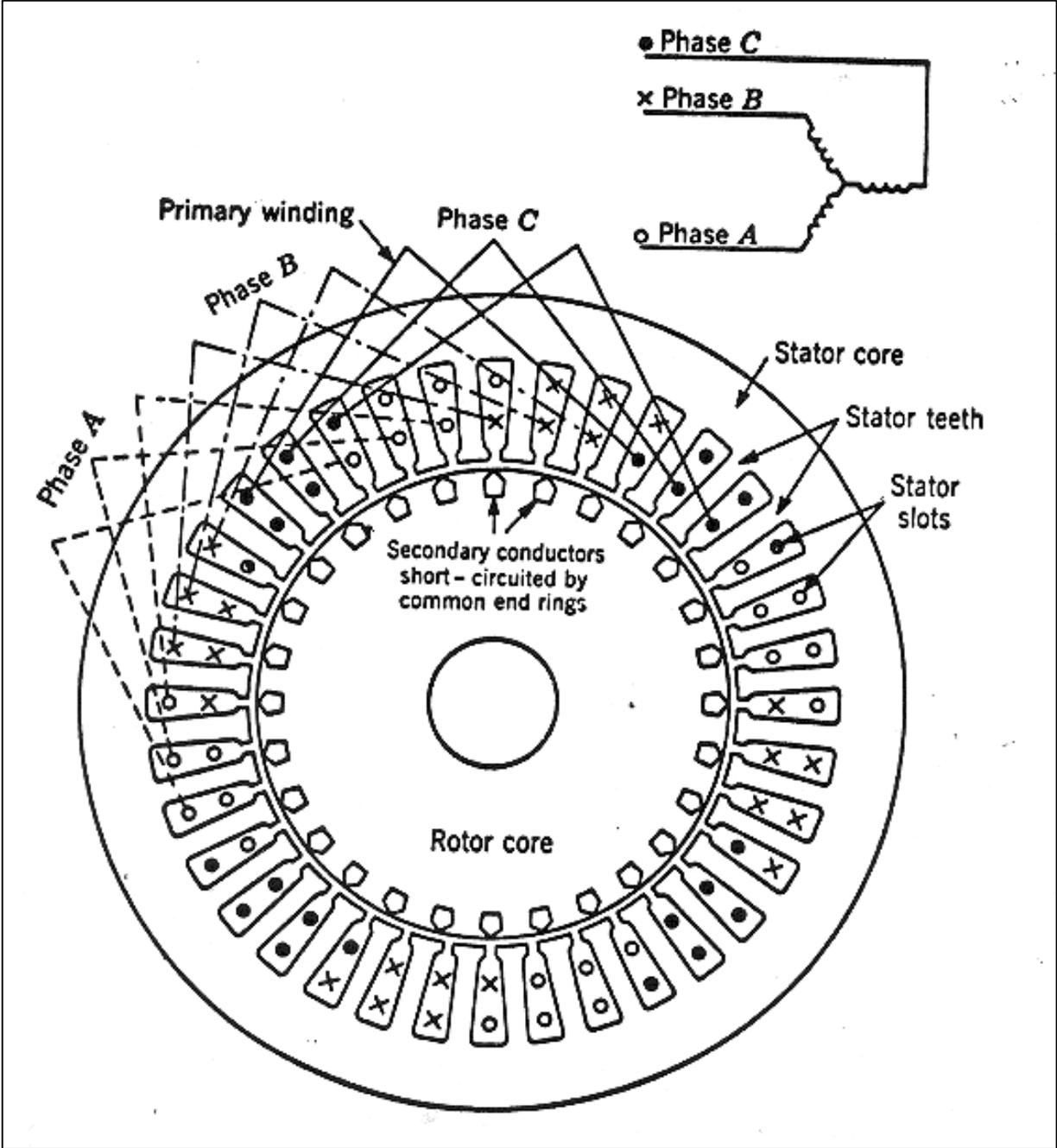
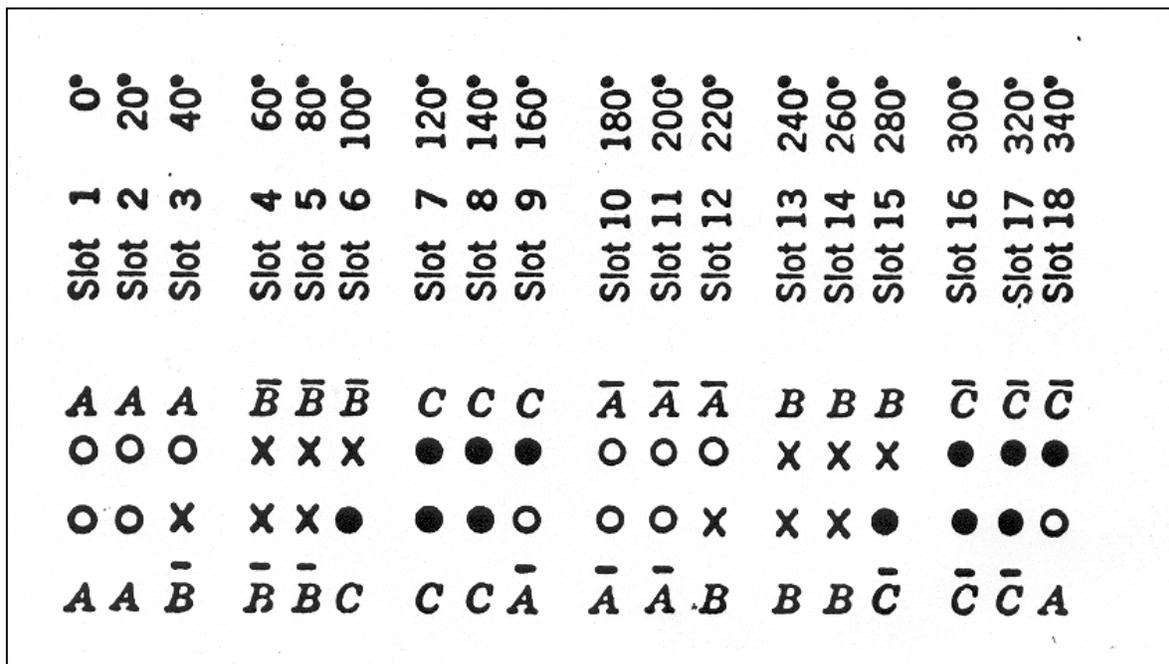


Figure-1 Cross-Section of Typical Three-Phase Induction Motor



**Figure-2 Double Layer, Two-Pole, Three Phase Winding: 60 degree Phase Belts, 8/9 Pitch**

Theoretical analyses of the electromagnetic fields in motors have greatly advanced motor design. P.L Alger [6] derives from magnetic theory a system of equations for calculation the magnetizing current of the induction motor. These equations are taken from the General Equivalent Circuit, due to Steinmetz [5], which describes the transformer or the induction motor when the magnetizing reactance is assumed to be the secondary. A simplified equivalent circuit for the induction motor in Figure 3, shows in some detail how the principle 5th and 7th phase belt, harmonic leakage reactances add to the fundamental motor reactance. The addition of the higher order slot harmonics is also shown in Fig. 3. As will be shown, the rotation harmonic magnetic fields degrade the motor performance. The detailed description of the Equivalent Circuit in Fig. 3 is given in Alger's definitive text, "The Nature of Induction Machines" [6].

Although the polyphase induction motor can be easily visualized by the phasor or circle diagram, it is not convenient to make multiple, repetitious exact calculations by this graphical method. The magnetic and electrical forces in the motor can be precisely modeled by computer using the Equivalent Circuit in Fig. 3. The Equivalent Circuit provides the model for summing the effects of the several rotating magnetic fields. Alger and others have considered each rotating field to be a separate motor connected to a common shaft. From Fig.3, it is seen that the 5th and 7th harmonics are major sources of reactance leakage and that reduction of these effects result in major improvements in motor efficiency.

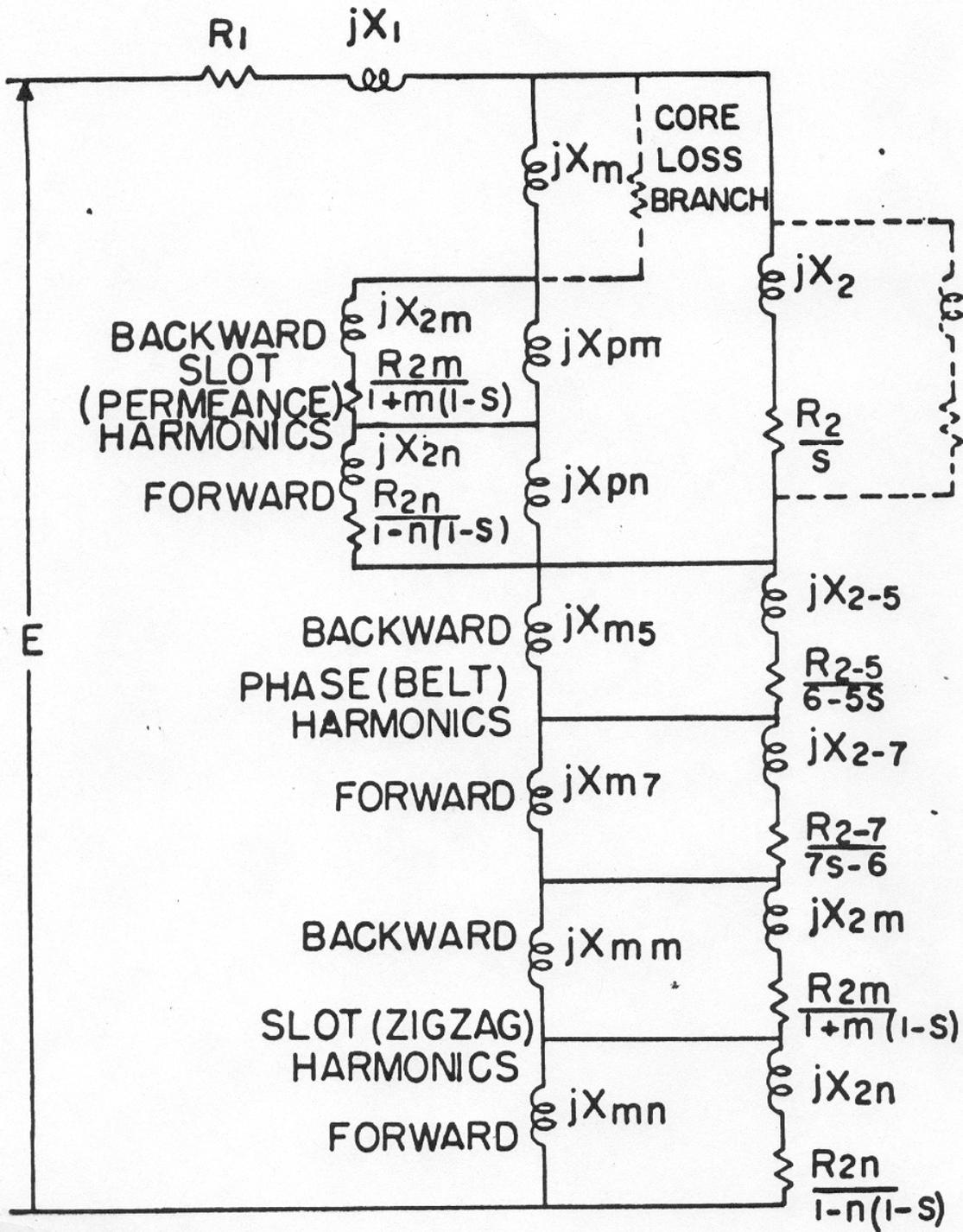


Figure-3 General Equivalent Circuit of a Polyphase Induction Motor

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Where the constants are given by:

E	=	Impressed voltage (volts) = line voltage / root 3 for 3-phase "Y" motors.
R1	=	Primary resistance (ohms).
R2	=	Secondary resistance (ohms).
X1	=	Primary leakage reactance (ohms).
X2	=	Secondary leakage reactance (ohms).
Xm	=	Magnetizing reactance (ohms).
Xm5	=	Magnetizing reactance of 5th harmonic field (ohms).
s	=	Per unit slip (expressed as fraction of synchronous speed).
j	=	Imaginary operator.

All the n-harmonic fields, Xm5, 7, 11, 13, 17..., rotate at speeds, 1/n, which are slower than the fundamental field, and so they will induce parasitic, non-useful voltages when the motor is at the synchronous speed. They are all part of the leakage reactance, X2, and create a continually changing shape of the flux wave as it revolves. The total phase belt, leakage reactance is the sum of all the reactance due to these phase belt harmonics. Alger (ibid.) in equations 7.67 and 7.68 shows that the reactance contribution from the harmonic magnetic fields decreases with the 4th power of the harmonic order. Therefore, reactance contributions higher than the 12th harmonic become negligible.

Using the Fourier method of analysis, the air gap magnetomotive force distribution may be represented as the sum of a series of sine waves. Alger shows that for each harmonic order there is a forward and backward rotating field. There are both even and odd harmonic fields in this series. The even harmonic fields represent the dissymmetry in the field and cancel out when there are an integral number of phase belts per pole. The rotation of ascending orders of odd harmonic fields alternates between forward and backward (5th = Backwards, 7th = Forwards, etc.) because of the mathematics of the Fourier Series. Also, the triplen harmonic orders (multiples of 3) cancel out in balanced 3-phase systems. Accordingly, we find very small even and triplen harmonics in the motor harmonic data, see Fig. 6, where the odd harmonics, minus the triplens, are alternating, B and F, with increasing order.

## II. Sources of Motor Harmonics

In Fig. 2, considering the top coils only, the first three coils are connected in series to form a phase belt for phase A. The next three are connected in series in phase belt B, reversed, and the next three similarly are in phase belt C. The next 9 coils repeat this sequence with the current flow reversed. There are in all, 6-phase belts in 2 pole pitches: each pole is 3 slots wide, or 60 degrees. The bottom coils are exactly similar but are displaced 8 slots or 160 degrees from the top coils and are reversed in current flow, giving a net displacement of  $180 / 9 = 20$  degrees. If the winding had 100% pitch, the top and bottom coils would carry identical in-phase currents and the magnetic field would not rotate. In Fig. 2, in order to generate the rotating field, 2 of the

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3 windings in each phase belt carry coils of the same phase winding and 1st winding carries coils of a different phase. This common 60-degree phase belt, 8 / 9 pitch, stator winding generates the rotating magnetic field, but also, because of the required geometry, with the magnetic interaction of the inductive windings, these stators generate severe 5th and 7th current harmonics, which must be controlled for good motor performance. There are also higher order harmonics associated with the number of stator and rotor slots and with the rotor design, however, these are usually of lower magnitude and are less important in motor design and performance.

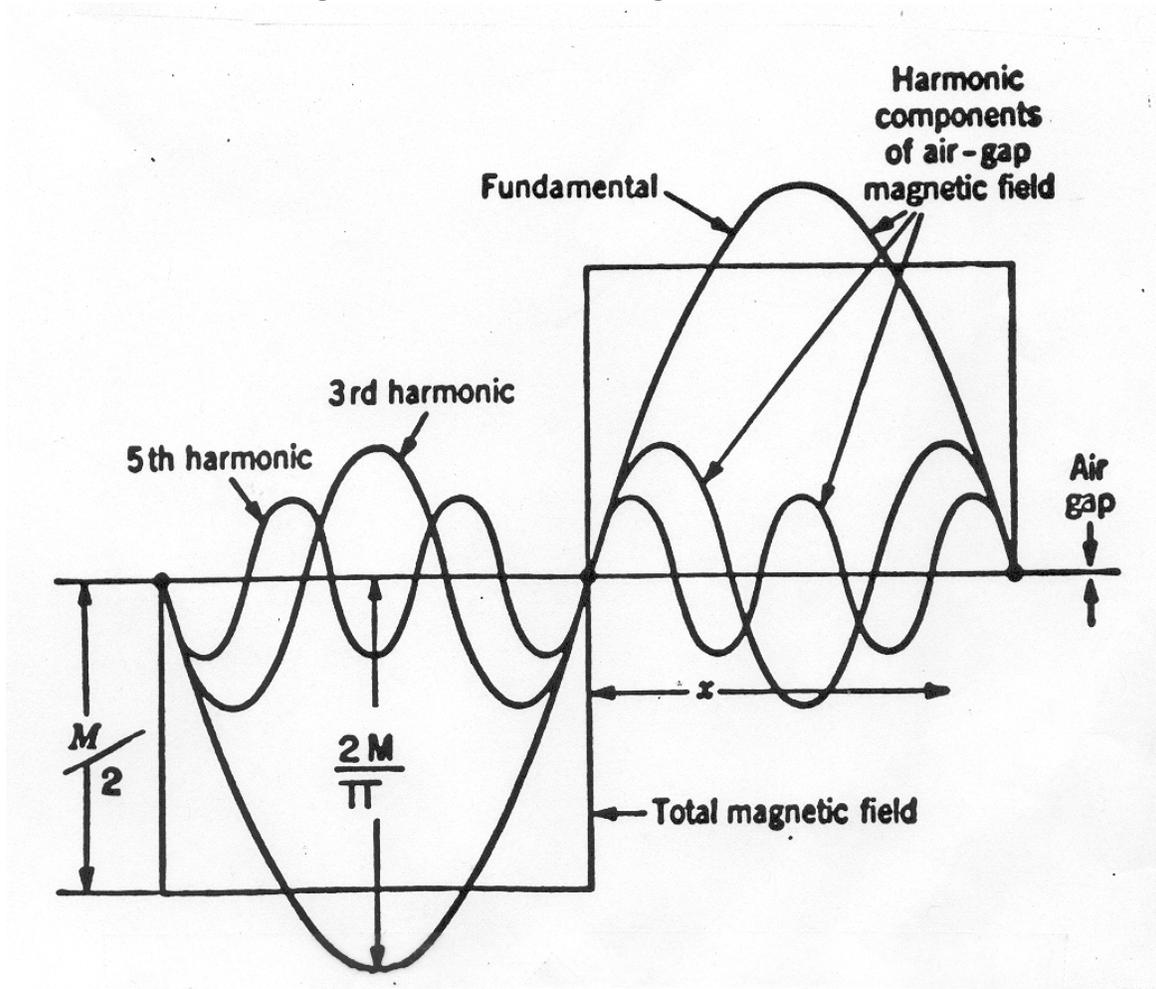
When calculating the voltage induced into the rotor by the magnetic AC fields of the stator coils which are arranged in phase belts, corrections must be added to allow for the non-linear coil distribution,  $Kd$ , and for the non-unity coil pitch,  $Kp$ . These factors are included in the angular velocity form of Faraday's Law (3) to calculate the voltage induced into the closed rotor circuit by the fundamental frequency magnetic wave.

$$e = 4.44 Kp Kd Nst f B \quad (3)$$

where  $e$  is induced voltage,  $Kp$  is the pitch factor,  $Kd$  is the distribution factor,  $Nst$  is the number of turns in the stator coil,  $f$  is the frequency in cps (cycles per second) and  $B$  is the field strength. Equation (3) can also be used to calculate the harmonic voltages induced in the rotor. Alger [6] and Cochran [7] give numeric equations to calculate the value on  $Kp$  and  $Kd$ , which vary for each harmonic. Calculation of the induced harmonic voltages and of the values of  $Kp$  and  $Kd$  for each harmonic has been of interest to motor design engineers, because, by controlling the values of  $Kp$  and  $Kd$  through varying the different arrangements of slots, poles and coil pitch, it is possible to slightly reduce certain undesirable harmonic fields and to improve motor performance. These marginal design considerations were the only methods available of controlling stator space harmonics and they have become imbedded in accepted stator winding design which is used in all modern induction motors.

To support the design engineer, and to develop equations for the harmonic stator rotating magnetic fields, an ideal coil was considered. When AC current is applied to this coil, geometric calculation yields a simple Fourier series of  $n$  trigonometric terms of alternating sign and of decreasing magnitude. For each harmonic term, two opposing harmonic fields are described: one rotating forward (F), that is, in the direction of the fundamental wave, and the other rotating backwards (B). When series-connected coils of the same phase are nested together, in the resultant magnetic field for each harmonic, the magnitude of the F and B fields within the odd harmonic fields add, while those within the even harmonic fields cancel. Therefore, in the observed stator fields, the even harmonics are much smaller and the odd harmonics dominate. Also, when 3-phase windings are considered, in the resultant field, the third harmonic and all higher  $3n$  harmonics also cancel. Therefore, all triple harmonics are also reduced. The only remaining stator harmonics in 3-phase induction motors are: 5B, 7F, 11B and 13F. The higher order harmonic fields are usually neglected. In Fig. 4, the principle harmonics through 5 are

superimposed on the fundamental sine wave. This mathematically predicted series of harmonics is observed in stator winding fields as shown in see Fig. 6 and Table 1.



**Figure-4. Harmonics of Magnetomotive Force (MMF) Wave Superimposed on the Fundamental**

Within the motor, harmonic MMF waves are said to exhibit space harmonics because these harmonics have geometric dimension, whereas the voltage and amperage harmonics are said to have time harmonics, because these harmonics are resolved in the time-dependent equations. Within the motor, space and time harmonics superimposed on the fundamental frequency sine wave fields, cause many harmonic forces and currents. For the purposes of analysis, Alger and others have considered these harmonic fields to be separate motors operating on the same shaft. The harmonic fields rotate at fractional speeds,  $1/n$ , where  $n$  is the order of the harmonic, and cause unexpected effects on the speed-torque curve; as shown in Fig. 5. After 1910, this deviation from theory stimulated great interest across Europe and the USA, which led to many significant technical papers being published by Punga [8], Dreese [9], Stiel [10], Graham [11], Kron [12], Weber and Lee [13], Trickey [14], Richter [15], Alger [16] and others. Between 1910 and 1950 a thorough understanding of the theory and behavior of the motor was

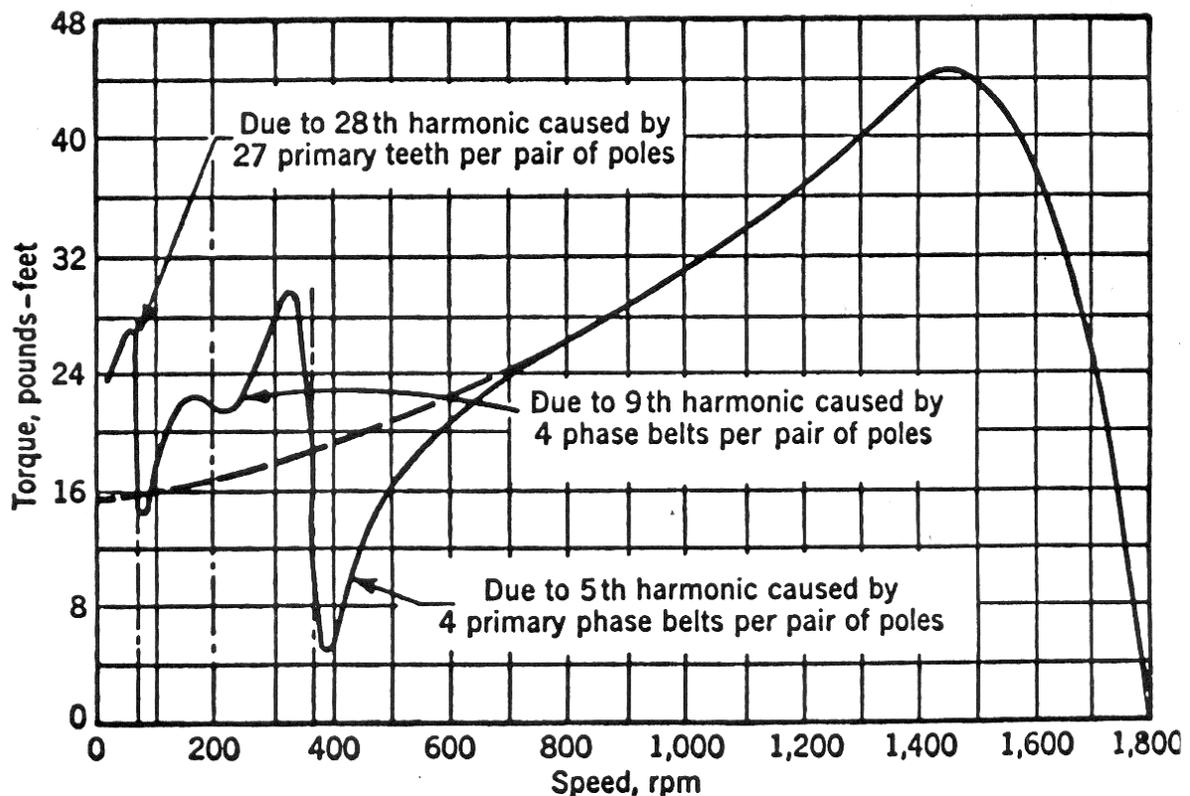
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developed, as reviewed by Alger and by Cochran. Since these older engineers have now retired, not many improvements have been made on their analyses and designs. At this time, with the introduction of a new method of harmonic control, the USES Phase Balancing Circuit, the Computer Aided Design (CAD) process holds promise for renewed design efforts that will optimize the stator design by eliminating the embedded, but less necessary, harmonics control features. A lighter, less expensive, more efficient motor will then evolve.

### III. Effects of Stator Harmonic Fields

The stator harmonic fields produced by the windings and the slots are of great importance to the designer who must keep them under control as a condition of good motor performance. The motor must rotate at one speed, which is close to the synchronous speed of the fundamental stator wave. All other harmonic fields and forces at lower speeds of rotation degrade the motor performance. The most important harmonic-related effects are of 5 kinds:

1. Asynchronous Crawling is caused by the space harmonics in the air gap, as shown in Fig. 5, which induce rotating voltage fields in the rotor, Alger [16]. These harmonic voltage fields cause currents, and produce torques similar to those of the fundamental, but they have more poles and therefore, lower synchronous speeds. Asynchronous crawling is caused by 5th and 7th harmonic fields that rotate at reduced speeds  $1/5$ th and  $1/7$ th of the synchronous speed of the fundamental. When the rotor from start accelerates through the synchronous speed of one of these harmonics, its torque reverses, causing a dip in the torque-speed curve shown in Fig. 5. Unless minimized by good design or other harmonic-reducing measures, asynchronous crawling seriously impairs motor starting ability. For the forward-rotating harmonics fields, 7, 13, 19, the harmonic rotor torque falls to zero at the synchronous speed and at higher speeds, it reverses, becoming a brake on the rotor. For the backward-rotating harmonic fields, 5, 11, 19, their synchronous speeds are reached when the motor is driven backwards at  $1/n$  speed, thus at all forward rotation they are a brake on the rotor. Therefore, the 5th and 7th harmonic fields exert a braking torque on the rotor at all speeds above  $1/5$ th and  $1/7$ th of the synchronous speed of the motor. At standstill, the net torque due to all the harmonics is normally negative, because the backward rotating fields having fewer poles than the forward rotating fields produce more braking torque than the accelerating torque produced by the accelerating harmonics. Also, the braking torque is larger because the B fields, with fewer poles, are of greater magnitude in the Fourier series. Both to design a self-starting motor and to increase the torque for a given mass of winding copper, reducing the 5th and 7th harmonic fields by varying the geometry of the stator field windings had been the goal of intense research effort. This design competition has been intense, with advantage shifting on improvements of a fraction of a percent.



**Figure-5 Asynchronous Crawling of Induction Motor**

2. Standstill Locking is present in all motors because the stator and rotor windings are in discrete coils or bars around the circumference of the air gap, and if the rotor is turned slowly with voltage impressed on the stator, for many slot variations, there will be a cogging or change in motor torque as it turns, Alger [6]. Some locking tendency, which is a field geometry effect, is present in every motor for the fundamental and for each harmonic field present. Locking is reduced by using fractional-pitch windings, preferably  $5/6$  for a 60-degree phase belt winding, as shown in Fig. 5, and by improved rotor design.

3. Synchronous Crawling is caused when the synchronous speeds of the harmonic fields interact, Alger [16]. As the rotor speed increases, so do the speeds of the harmonic fields caused by the rotor. At the speed when the backward rotating stator field, 5th or 11th, is rotating forward at the same speed as a forward rotating field with the same number of poles, the two fields will lock into step and the motor torque is reduced. This locking torque is caused by the stator harmonic fields and can be controlled by varying the relative number of poles in the rotor and stator design. When severe, synchronous crawl can exceed the torque of the fundamental and can sometimes be used for special effect motors like solenoids.

4. Magnetic noise and vibration are caused when two stator harmonic fields, with numbers of poles differing by 2, coexist in the air gap. Under these conditions they will produce unbalanced, radial magnetic forces, which vibrate the motor as a whole. Also, symmetrical radial

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forces of high frequency caused by superposition of rotating magnetic fields of different pole numbers, result in stator vibration and magnetic noise, Alger [16].

5. Voltage ripples are caused by stator harmonic fields, principally the 5th and 7th, which induce voltages and currents in the rotor. These terminal voltage ripples cause additional core losses and produce high frequency ripples in the supply lines and inductive interference in communication circuits.

Thus, we see that of the 5 most troublesome harmonic effects in induction motors are dependent on the 5th and 7th stator fields. All these retarding effects are caused by these principle stator fields and would be eliminated if these fields could be reduced. Past efforts to minimize these effects of the stator fields have during the period from 1920-1950 caused many design features to become embedded into the accepted stator winding design configuration. In the last section of this paper, we will present a new method of controlling the 5th and 7th stator fields, which was not available during earlier periods of intense research. Therefore, in motors that can now be phase balanced by the USES Circuit, the 5th and 7th harmonics will be significantly reduced, and if the basic stator winding design is now optimized by Computer Aided Design to eliminate the unnecessary harmonic control features, then a more efficient motor design will result.

### **III. NEW MOTOR HARMONICS DATA**

Having theoretically described the sources of motor harmonic fields and shown their causes and effects, we now present new experimental data from the Cutler-Hammer measurements [1] of a 150 HP motor at the Sykesville Lab of Northrop Grumman Corp. These data are part of the Power Study of this Motor-USES Circuit used by Emmet and Ray [2,3] to develop the theory of phase balancing that improves power factor, attenuates transients and compensates surges. These harmonic data were recorded and analyzed for each phase at 4 motor power levels, 50%, 75%, 90% and 100% RLA. The typical data are shown here for the A Phase at 75%, in Table 1 and Fig.6. Emmet and Ray show that the USES Unit reduces the total distortion of amperage harmonics, ITHD, of the motor circuit, by 30%. Considering the individual harmonic values in the harmonic spectrum, Table 1 and Fig. 6 show that the USES Unit also decreases the dominant 5th harmonic of the stator field by 47%. Although there is a slight increase in the higher harmonics, 17th-23rd, their effect on motor performance may be neglected because of the much smaller absolute magnitude of these field vectors and the much slower harmonic field rotation. The phase balancing reduces the 5th and 7th amperage harmonics produced by the stator and thereby improves the motor performance.

Table 1

Cutler-Hammer Motor Harmonics Data for 75 RLA

Harm. Order	ON	OFF	Harm. Order	ON	OFF
2	0.165	0.142	14	0.043	0.000
3	1.055	0.918	15	0.033	0.000
4	0.066	0.057	16	0.000	0.000
5	1.879	3.414	17	0.791	0.142
6	0.033	0.018	18	0.066	0.000
7	1.714	2.134	19	0.264	0.057
8	0.000	0.028	20	0.033	0.000
9	0.033	0.057	21	0.066	0.028
10	0.033	0.026	22	0.033	0.000
11	0.396	0.512	23	0.165	0.085
12	0.000	0.000	24	0.000	0.000
13	0.231	0.142	25	0.033	0.026

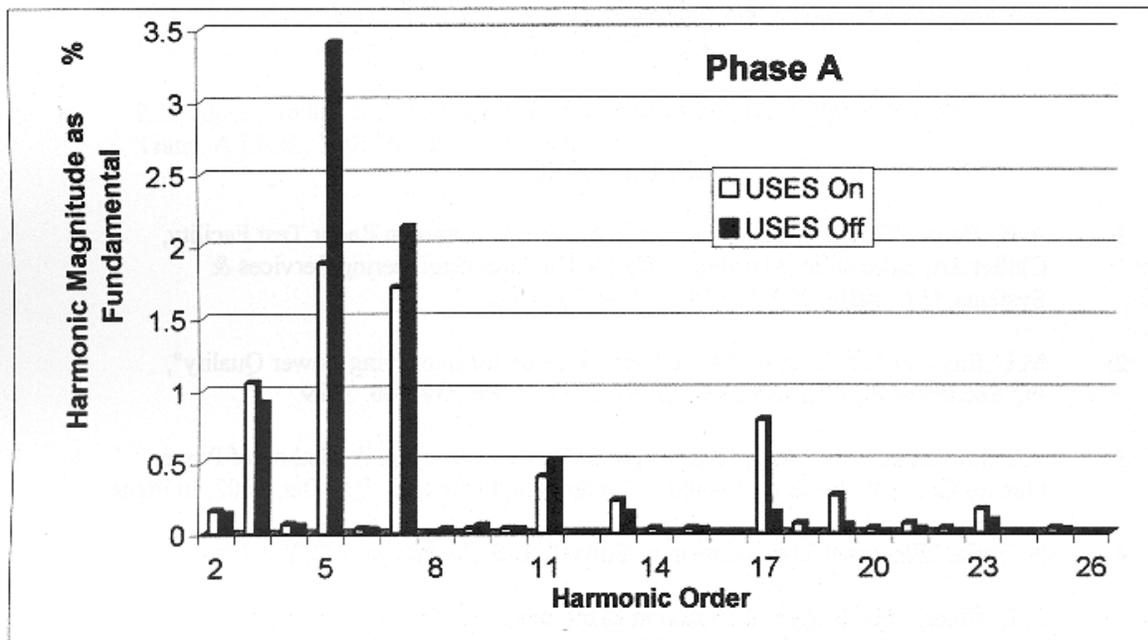


Figure-6. Motor Harmonics for Phase A at 75 RLA with USES on/off.

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## SUMMARY

These new data show that by balancing the phases with the USES wrap-around magnetic chokes, eliminates by half the backward rotating 5th order stator harmonic field, reduces by 20% the 7th harmonic. Some harmonic energy is shifted above  $n=15$ , which do not effect motor performance. The consequent efficiency increase provides major opportunities for advances of induction motor design and motor efficiency, which were not possible in the hay-day of motor research. We suggest that this 50% reduction of the 5th harmonic in induction motor circuits is responsible for most of the 10 % increase of motor efficiency shown in [3]. This new capacity to significantly reduce the backward rotating 5th harmonic without distorting the factors,  $K_p$  and  $K_d$ , should allow motor design to be reviewed and optimized. Installation of the USES Unit in the existing power distribution circuits parallel to the motors, should provide major additional benefits, such as phase balancing, power factor correction, transient protection and surge absorption, in addition to increasing motor efficiency and performance characteristics by controlling the 5th harmonic of the stator field. A new generation of improved motors can now be designed by eliminating the harmonic-suppression stator windings and incorporating the USES phase-balancing circuit in a new stator design.

We welcome your comments and questions.

R.T. Emmet and M.L. Ray

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